Hausdorff School

"PDEs in Fluid Mechanics"

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organized by
Diego Alonso-Orán, Juan J. L. Velázquez

Abstracts

Maria Colombo (EPFL Lausanne [Online])

Lecture: Nonuniqueness results from 2D Euler equations to 3D Navier-Stokes equations

Abstract: In his seminal work, Leray demonstrated the existence of global weak solutions, with nonincreasing energy, to the Navier-Stokes equations in three dimensions. In this course we will introduce the problem and aim at exhibiting two distinct Leray solutions with zero initial velocity and identical body force. The starting point of our construction is Vishik's answer to another long-standing problem in fluid dynamics, namely whether the Yudovich uniqueness result for the 2D Euler system can be extended to the class of L^p-integrable vorticity. Building on Vishik's work, we construct a 'background' solution which is unstable for the 3D Navier-Stokes dynamics in similarity variables; the second solution from the same initial datum is a trajectory on the unstable manifold associated to the background solution, in accordance with the predictions of Jia and Sverak.

Ángel Castro (Instituto de Ciencias Matemáticas, ICMAT)

Lecture: Traveling waves close to the Couette flow

Abstract: In this talk we shall study the existence of smooth traveling waves close to the Couette flow for the 2D incompressible Euler equation for an ideal fluid. It is well known that this kind of solution does not exist arbitrarily close to the Couette flow if the distance is measured in $H^{\frac{3}{2}+}$. In this course we will deal with the case $H^{\frac{3}{2}+}$.

David Lannes (CNRS-Institute de Mathématiques de Bordeaux)

Lecture: Wave-structure interactions

Abstract: The course is divided as follows:

- **Lecture 1:** modelling Water-waves are classically described by the Euler equations with a free surface and a constrained pressure at the surface (the pressure at the surface is equal to the atmospheric pressure, assumed to be constant). In some particular configurations (shallow water for instance), the Euler equations can be replaced by simpler asymptotic models. Let us now consider the presence of a partially immersed object: under this object, the surface is no longer free (the surface of the water coincides with the bottom of the object), but the pressure at the surface is now free and can be understood as the Lagrange multiplier associated with the constraint on the surface elevation. Its value represents the pressure exerted by the fluid on the object. The modelling of wave-structure interactions consists in understanding the coupling between these two different situations. We will show how the problem can be reduced in some cases to a transmission problem coupled with some scalar ODEs.

- **Lecture 2:** the hyperbolic case The simplest wave-structure interaction model is obtained by implementing the above approach in the case where the waves are modeled by the hyperbolic nonlinear shallow water equations. After recalling some elements of the theory of hyperbolic initial boundary value problem we will analyse the corresponding transmission problem.

- **Lecture 3:** the dispersive case The nonlinear shallow water equations considered in Lecture 2 neglect the dispersive effects. The Boussinesq equations form a more precise approximation and take such effects into account. Because of the presence of dispersive terms, the hyperbolic tools used in the previous case become useless and specific tools must be introduced to handle the influence of dispersion on initial boundary value problems such as dispersive boundary layers, hidden trace regularity, etc.

Short talks

Rafael Granero-Belinchón (Universidad de Cantabria [Online])

Talk: On viscous surface waves

Abstract: In this talk we will review some recent results concerning the motion of a surface wave under gravity, capillary and viscous effects. We will consider the case of both odd and even viscosities and derive some asymptotic models. Finally, we will present some well-posedness theory for these models.

Claudia García (Universidad de Barcelona)

Talk: Time periodic solutions for the 3D quasi-geostrophic system

Abstract: In this talk, we aim to study time periodic solutions for the 3D inviscid quasi-geostrophic model. We show the existence of non trivial rotating patches by suitable perturbation of stationary solutions. Indeed, the construction of those special solutions are done through bifurcation theory from some revolution shape domains. In order to apply the bifurcation argument, one should study the linearized operator around the stationary solution. In this case, the spectral problem is very delicate and strongly depends on the shape of the initial stationary solutions. In fact, the spectral study can be related to an eigenvalue problem of a self-adjoint compact operator and we are able to implement

the bifurcation only from the largest eigenvalues of such operator which are simple. This is a joint work with T. Hmidi and J. Mateu.

Edoardo Bocchi (Universidad de Sevilla)

Talk: Rigorous thin film approximations of the one-phase unstable Muskat problem

Abstract: In this talk we deal with the one-phase Muskat problem driven by gravity and surface tension in the unstable case, with the fluid on top of a dry region. Considering the thin film regime, we derive two asymptotic approximations for this scenario. The lower order approximation is the classical thin film equation, while the higher order approximation provides a new refined thin film equation. We prove optimal order of convergence in the shallowness parameter to the original Muskat solutions for both models with low-regular initial data.

Gabriele Brüll (Lund University)

Talk: Waves of greatest height

Abstract: In this talk, I will present the phenomenon of so-called waves of greatest height for the fractional Korteweg–de Vries equation

$$u_t + uu_x + |D|^\alpha u_x = 0, \quad \alpha \in R,$$

where $|D|^\alpha$ is a Fourier multiplier operator with real symbol $|k|^\alpha$. For $\alpha = 2$ this is the classical Korteweg–de Vries equation, $\alpha = 1$ recovers the Benjamin–Ono equation, $\alpha = 0$ the Burgers' equation, $\alpha = -\frac{1}{2}$ is sometimes referred to as deep-water equation, and $\alpha = -2$ corresponds to the reduced Ostrovsky equation. In 1880 Stokes conjectured the existence of a traveling surface wave of greatest height–how he called it for the water-wave problem. He predicted a corner singularity at the crests of the periodic traveling wave, enclosing an angle of precisely 120°. Stokes' conjecture was affirmed about 100 years later by Plotnikov, Amick, Fraenkel, and Toland. Naturally, the question arises, whether water-wave model equations, such as some equations appearing in the fractional Korteweg–de Vries family, captured the phenomenon of Stokes' wave of greatest height. I will present some results concerning existence and regularity of such highest waves for the fractional Korteweg–de Vries equation for $\alpha < 0$. The talk is based on a joint work with R. Dhara (Brno).

Christina Lienstromberg (Bonn University)

Talk: Analysis of non-Newtonian Taylor–Couette flows

Abstract: I will offer an insight into mathematical models describing the dynamic behaviour of non-Newtonian thin-films flows in circular geometries. The resulting PDEs are in general nonlinear, degenerate, of fourth order, and with a possibly 'weak' dependence of the coefficients. I will discuss recent results on such evolution equations for the interface separating two viscous immiscible fluids, confined between two concentric cylinders rotating at a small relative velocity. In this so-called Taylor–Couette setting, two competing effects drive the dynamics of the interface – the surface tension and the shear stresses induced by the rotation of the cylinders. When the two effects are comparable, solutions behave, for large times, as in the Newtonian regime. For the regime in which surface tension effects dominate the stresses induced by the rotating cylinders, we prove global existence of positive weak solutions for both shear-thinning and shear-thickening fluids. In the case of a shear-thickening fluid, one observes that interfaces which are initially close to a circle converge to a circle in finite time.

In the shear-thinning case, we find that steady states are polynomially stable in the sense that, as time tends to infinity, interfaces which are initially close to a circle, converge to a circle at rate $1/t^{1/\beta}$ for some $\beta > 0$. The talk is based on joint works with Tania Pernas-Castaño and Juan Velázquez.